Overview of Drell-Yan Physics

Theory

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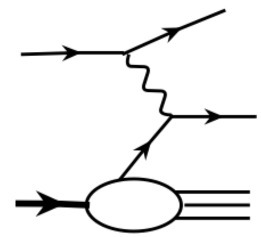
Outline

- ❑ Drell-Yan mechanism in parton model
- ❑ Drell-Yan mechanism in QCD
- ❑ QCD factorization for inclusive Drell-Yan
- ❑ Collinear vs TMD factorization
- ❑ The sign change
- ❑ Drell-Yan offers much more than the sign change
- ❑ Summary and outlook

Feynman's parton model

❑ Parton model for inclusive DIS - unpolarized:

$$\sigma^{\rm DIS}_{\ell+h\to\ell+X}(x_B,Q^2) = \int dx\ \phi_{{\rm parton}/h}(x)\ \hat{\sigma}^{\rm Elastic}_{\ell+{\rm parton}\to\ell+X}(x_B/x,Q^2)$$
$$= \sigma_0 \sum_q e_q^2 \left[\phi_{q/h}(x_B) + \phi_{\bar{q}/h}(x_B)\right]$$
$$\sigma_0 = \sigma^{\rm Elastic}_{\ell+q\to\ell+q}(Q^2)$$



✧ **Prediction:**

- **Bjorken scaling**
- **Callan-Gross relation – parton has spin-1/2**

✧ **Predictive power:**

- **Universality of parton distribution:** $\phi_{{\rm parton}/h}(x)$

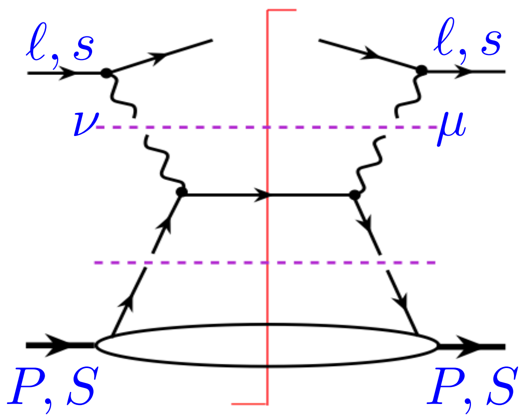
❑ Longitudinally polarized - A_{LL}:

$$\phi_{{\rm parton}/h}(x) \to \Delta\phi_{{\rm parton}/h}(x)$$

$$A_{LL} \propto \sum_q e_q^2 \left[\Delta\phi_{q/h}(x_B) + \Delta\phi_{\bar{q}/h}(x_B)\right] \Big/ \sum_q e_q^2 \left[\phi_{q/h}(x_B) + \phi_{\bar{q}/h}(x_B)\right]$$

Parton model is an approximation of QCD

❑ Leading order in QCD:



$$\Leftarrow \quad L^{\mu\nu}\, W_{\mu\nu}$$

$$\Leftarrow \quad \text{odd in } \gamma \quad \Rightarrow \quad \gamma^\alpha\,, \quad \gamma^\alpha\gamma^5$$

$$\Rightarrow \begin{cases} \langle P,S|\overline{\psi}(0)\,\dfrac{\gamma\cdot n}{P\cdot n}\,\psi_j(y)|P,S\rangle \\ \langle P,S|\overline{\psi}(0)\,\dfrac{\gamma\cdot n\,\gamma^5}{P\cdot n}\,\psi_j(y)|P,S\rangle \end{cases}$$

❑ Parity and Time-reversal:

$$\langle P,S|\overline{\psi}(0)\,\frac{\gamma\cdot n}{P\cdot n}\,\psi(yn)|P,S\rangle \quad \Longrightarrow \quad +\,\langle P,-S|\overline{\psi}(0)\,\frac{\gamma\cdot n}{P\cdot n}\,\psi(yn)|P,-S\rangle$$

$$\langle P,S|\overline{\psi}(0)\,\frac{\gamma\cdot n\,\gamma^5}{P\cdot n}\,\psi(yn)|P,S\rangle \quad \Longrightarrow \quad -\,\langle P,-S|\overline{\psi}(0)\,\frac{\gamma\cdot n\,\gamma^5}{P\cdot n}\,\psi(yn)|P,-S\rangle$$

❑ PDFs and Helicity distributions:

$$\phi(x) \propto \langle P,S|\overline{\psi}(0)\,\frac{\gamma\cdot n}{P\cdot n}\,\psi(yn)|P,S\rangle$$

$$\Delta\phi(x) \propto \langle P,S|\overline{\psi}(0)\,\frac{\gamma\cdot n\gamma^5}{P\cdot n}\,\psi(yn)|P,S\rangle$$

QCD is much richer!
Scaling violation
Role of gluon

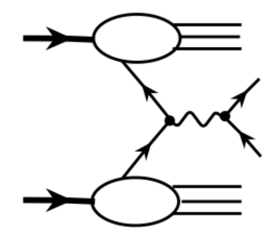
Drell-Yan mechanism in parton model

❑ Drell-Yan lepton-pair production:

$$\frac{d\sigma_{A+B\to\ell\bar{\ell}(Q^2)+X}}{dQ^2} = \sigma_0 \sum_q e_q^2 \int dx\, \phi_{q/A}(x) \int dx'\, \phi_{\bar{q}/B}(x')\, \delta(Q^2 - xx's_{AB}) + q \leftrightarrow \bar{q}$$

$$= \frac{\sigma_0}{s_{AB}} \sum_q e_q^2\, \mathcal{F}_{q\bar{q}}(\tau = Q^2/s_{AB}),$$

$$\sigma_0 = \sigma^{\mathrm{incl}}_{q\bar{q}\to\ell\bar{\ell}(Q^2)}$$



Effective flux: $\mathcal{F}_{q\bar{q}}(\tau) = \int dx\, \phi_{q/A}(x) \int dx'\, \phi_{\bar{q}/B}(x')\, \delta(\tau - xx') + q \leftrightarrow \bar{q}$

❑ Predictions:

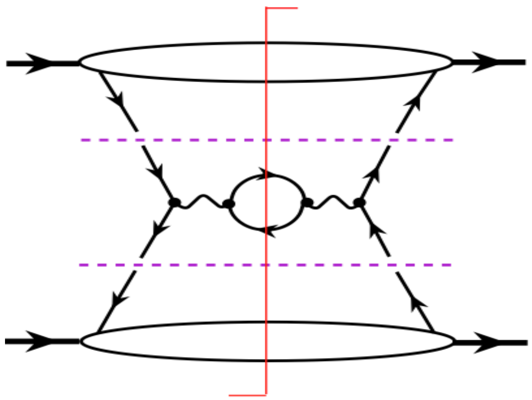
- ✧ No free parameter for production rate!
- ✧ Normalized Drell-Yan angular distribution

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\, d\Omega} = \frac{3}{4\pi}\left(\frac{1}{\lambda + 3}\right)\left[1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)\right]$$

- ✧ Transversely polarized virtual photon: $1 + \cos^2\theta$ distribution
- ✧ Lam-Tung relation: $1 - \lambda - 2\nu = 0$

Drell-Yan mechanism in QCD

❑ Leading order in QCD:



$\Leftarrow$ all γ structure: $\gamma^\alpha,\ \gamma^\alpha\gamma^5,\ \sigma^{\alpha\beta}(\text{or } \gamma^5\sigma^{\alpha\beta}),\ I,\ \gamma^5$

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❑ Parity and Time-reversal:

$$\langle P, S_\perp|\overline{\psi}(0)\,\frac{\gamma\cdot n\,\gamma_\perp^\sigma}{P\cdot n}\,\psi(yn)|P, S_\perp\rangle \quad\Longrightarrow\quad -\langle P, -S_\perp|\overline{\psi}(0)\,\frac{\gamma\cdot n\,\gamma_\perp^\sigma}{P\cdot n}\,\psi(yn)|P, -S_\perp\rangle$$

❑ transversity distribution:

$$h_1(x) \propto \langle P, S_\perp|\overline{\psi}(0)\,\frac{\gamma\cdot n\,\gamma_\perp^\sigma}{P\cdot n}\,\psi(yn)|P, S_\perp\rangle$$

❑ Asymmetries – collinear factorization:

$$A_{LL} \propto \sum_q e_q^2\,\Delta q(x)\Delta\bar{q}(x') \qquad A_{TT} \propto \sum_q e_q^2\,h_{1q}(x)h_{1\bar{q}}(x') \qquad A_L \propto \sum_q (c_v * c_a)\,\Delta q(x)\,\bar{q}(x')$$

$$A_N \propto \sum_q e_q^2\,T_q(x,x)\,\bar{q}(x') \qquad A_{LT} \propto \sum_q e_q^2\,\Delta q(x)\,\tilde{T}_{\bar{q}}(x')$$

From parton model to QCD

- **Parton model – big K-factor:**

$$K \equiv \frac{(d\sigma/dQ^2)_{\mathrm{PM}}}{(d\sigma/dQ^2)_{\mathrm{exp}}} \gtrsim 2$$

  - ✧ **Parton model = leading order QCD without DGLAP evolution**
  - ✧ **Leading order QCD calculation has the same size K-factor**

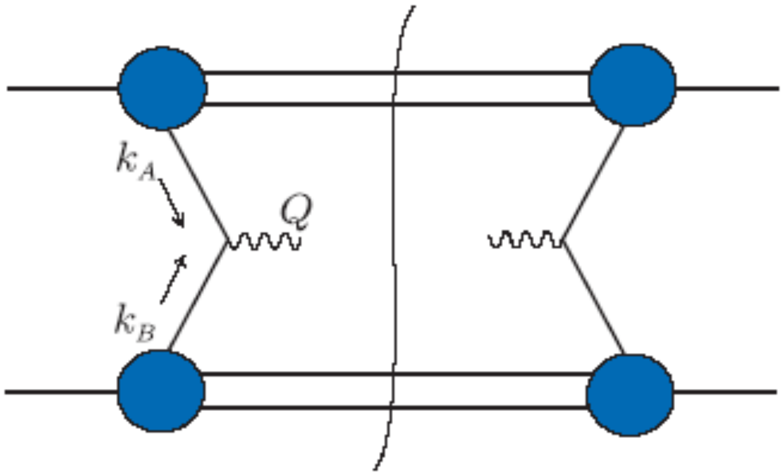
- **QCD calculation at NLO and higher:**

$$K \equiv \frac{(d\sigma/dQ^2)_{\mathrm{NLO}}}{(d\sigma/dQ^2)_{\mathrm{exp}}} = 1$$

  - ✧ **Normalization uncertainty in QCD global fit is limited by systematic error of individual experiment**
  - ✧ **High order corrections are sensitive to if the virtual photon's invariant mass is space-like or time-like**

$$\log(q^2_{\mathrm{DIS}}) \rightarrow \log(-q^2_{\mathrm{DIS}}) + \log(-1)$$

Why Drell-Yan factorization make sense?



- Pinch of active parton momenta
- Long-lived partonic states
- lowest order kinematics determines the process

$$\int d^4k_A \frac{1}{k_A^2 + i\varepsilon} \frac{1}{k_A^2 - i\varepsilon} \to \infty$$

$$\frac{d\sigma}{dQ^2 dy} = \int dk_{A,T}\, dk_{B,T} dk_A^- dk_B^+ \; H_{\mu,\nu}(Q^+, Q^-, k_{A,T} + k_{B,T})$$
$$\times \mathrm{Tr}\{\gamma^\mu \Phi_A(Q^+ - \cancel{k_B^+}, k_{A,T}, k_A^-)\gamma^\nu \Phi_B(k_B^+, k_{A,T}, Q^- - \cancel{k_A^-})\}$$

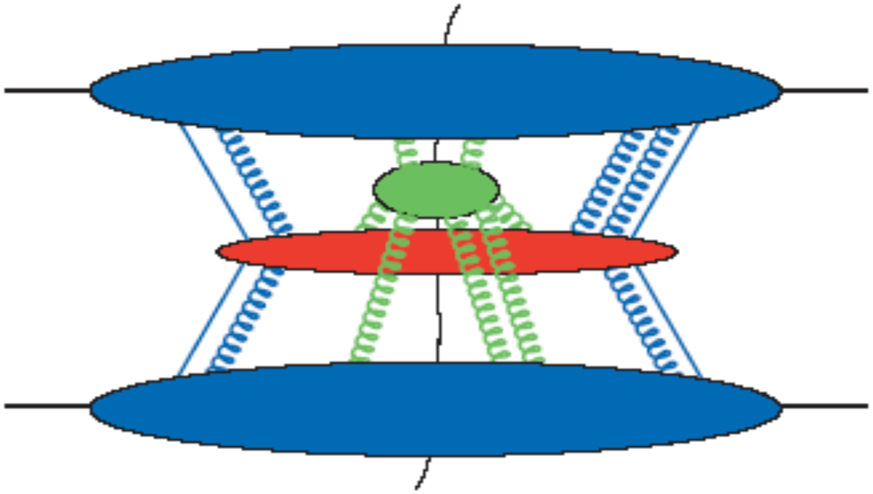
Approximation:

$$k_{A,T}^2\ ,\ k_{B,T}^2 \ll Q^2$$
$$k_A^- \ll Q^-$$
$$k_B^+ \ll Q^+$$

$\Longrightarrow$ **Drell-Yan formula**

QCD dynamics is rich and complicate

❑ Leading pinch surface:



Analysis of leading (pinch or singular) integration regions gives the following:

Hard (Large P_T or way off shell)

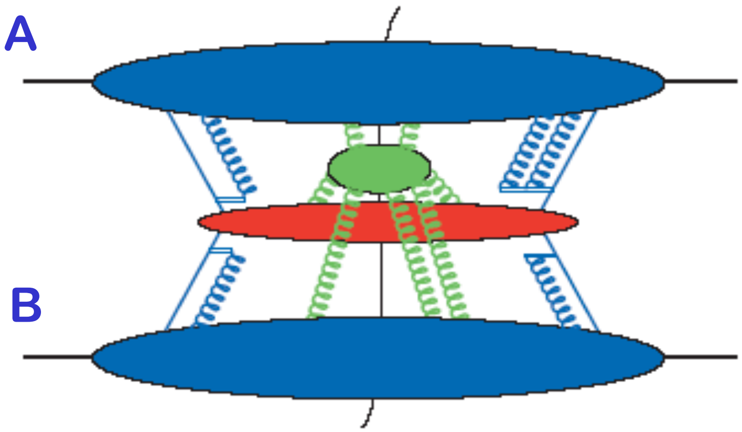
Collinear (to A or to B, small P_T) – one-pair "physical parton" from each hadron

Soft (All components small, includes "Glauber.")

❑ Factorization:

Long-distance distributions are process independent

Eikonalization of collinear gluons



- ❑ **Extra gluon is trouble:**
  - ✧ Factorization means one active parton from each beam hadron
  - ✧ Colored quark always has longitudinally polarized gluons

- ❑ **But, collinear gluons are ok:**
  - ✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$
  - ✧ The sum of the effect can be represented by the eikonal lines

If hadron A moving along "+", hadron B moving along "-"

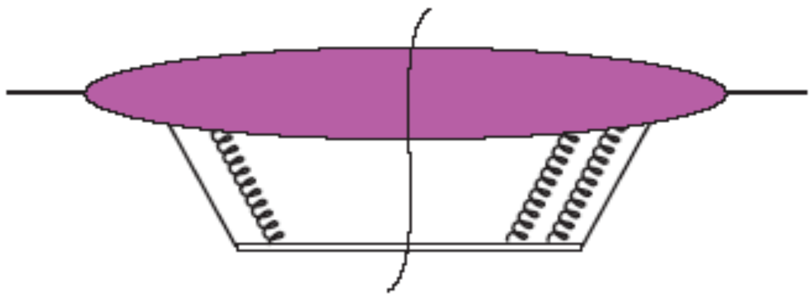
The direction of eikonal line "u" for A is "-", and for B is "+"

Propagator: $$\frac{i}{k \cdot u + i\varepsilon}$$

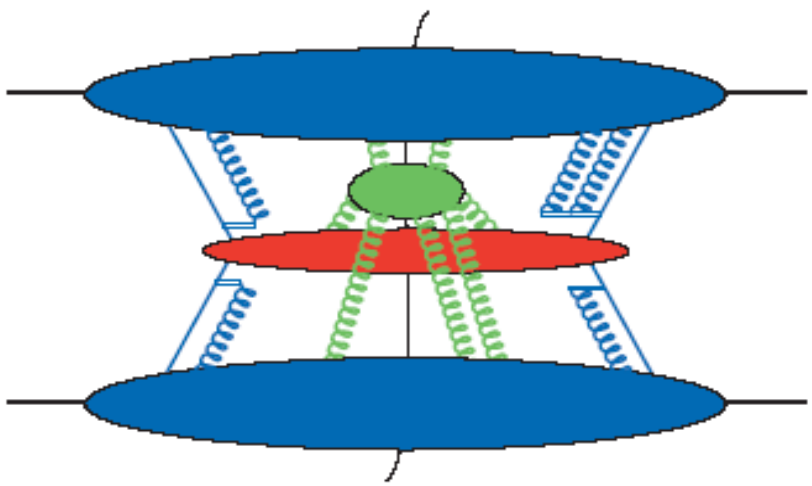
Vertex: $-i\, g\, t^a\, u^\mu$ with SU(3) color generator t^a

Factorization of PDFs

Parton distribution in diagrams

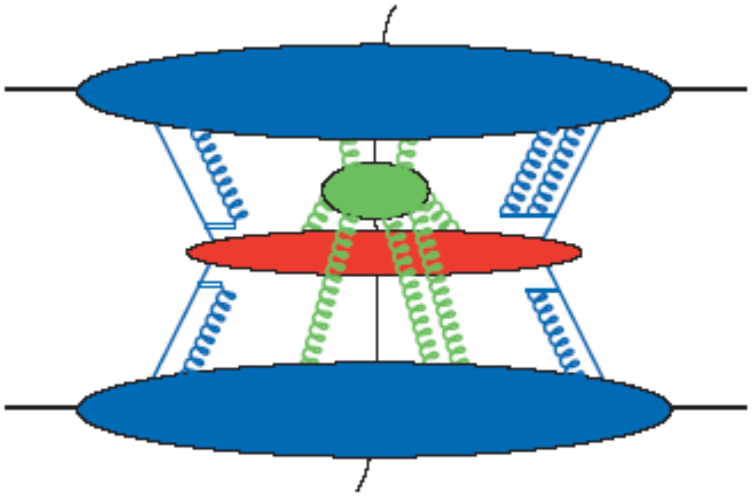


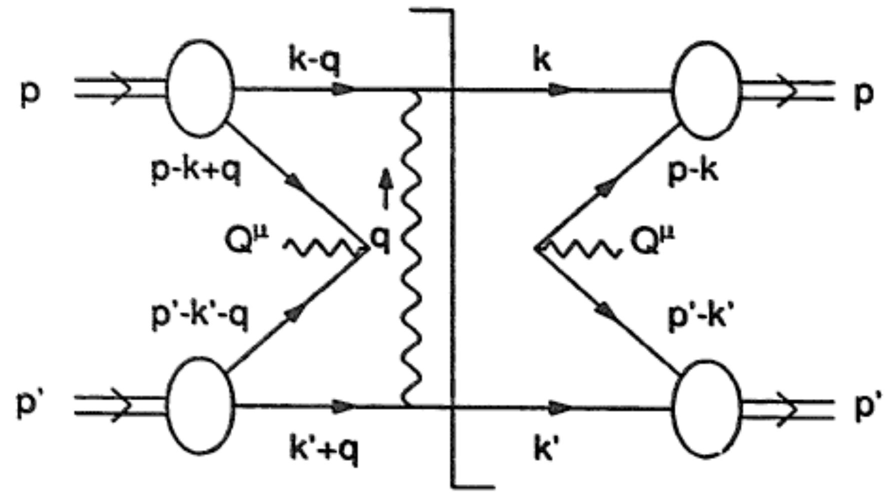
Compare



Need to get rid of the soft gluons!

Trouble with the soft gluons



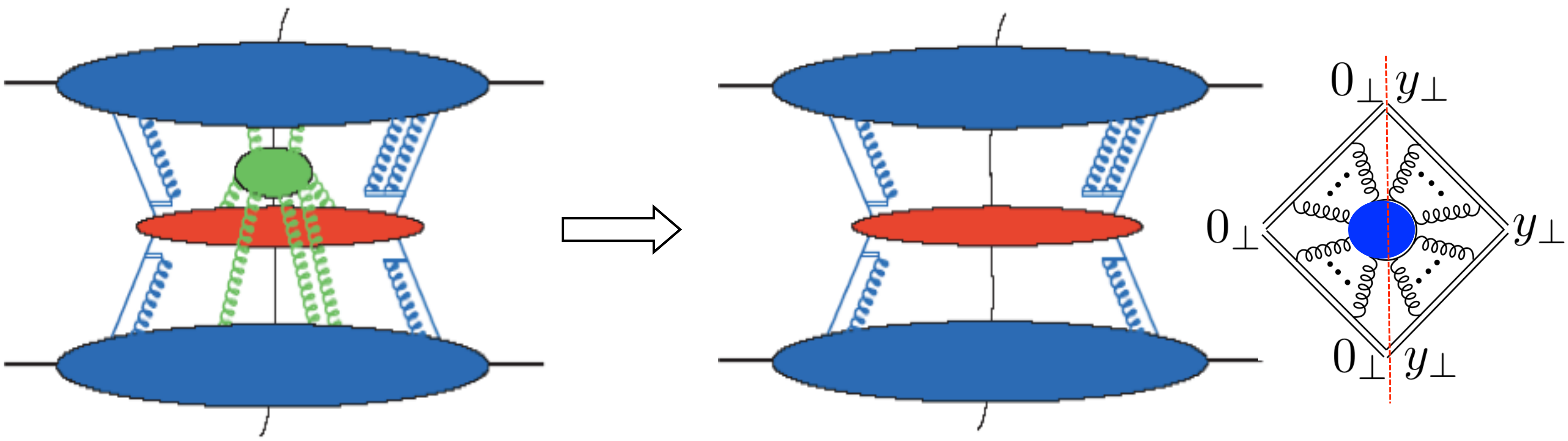


✧ Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B

✧ The soft gluon approximations (with the eikonal lines) need $q^{\pm}$ not too small. But, $q^{\pm}$ could be trapped in "too small" region due to

Pinch from spectator interaction: $q^{\pm} \sim M^2/Q \ll q_{\perp} \sim M$

Soft gluons take care of themselves



- Most technical part of the factorization
- Sum over all final states to remove all poles in one-half plane

  – no more pinch poles
- Deform the $q^{\pm}$ integration out of the trapped soft region
- Eikonal approximation ⟹ soft gluons to eikonal lines – gauge links
- Collinear factorization: unitarity ⟹ soft factor = 1

Factorized Drell-Yan cross section

❑ TMD factorization ($q_\perp \ll Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$
$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}}\, e^{y} \qquad x_B = \frac{Q}{\sqrt{s}}\, e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

❑ Collinear factorization ($q_\perp \sim Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\; \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu)$$

❑ Spin dependence:

The factorization arguments are independent of the spin states of the colliding hadrons

⟹ same formula with different distributions for γ^*,W/Z, H^0…

TMD vs collinear factorization

- **TMD factorization and collinear factorization cover different regions of kinematics:**

  Collinear: $Q_1 \ldots Q_n \gg \Lambda_{QCD}$

  TMD: $Q_1 \gg Q_2 \sim \Lambda_{QCD}$

  - ✧ One complements the other, but, cannot replace the other!
  - ✧ Predictive power of both formalisms relies on the validity of their own factorization

  Consistency check – overlap region – perturbative region

- **"Formal" operator relation between TMD distributions and collinear factorized distributions:**

  spin-averaged: $$\int d^2k_\perp \, \Phi_f^{\mathrm{SIDIS}}(x,k_\perp) + \mathrm{UVCT}(\mu_F^2) = \phi_f(x,\mu_F^2)$$

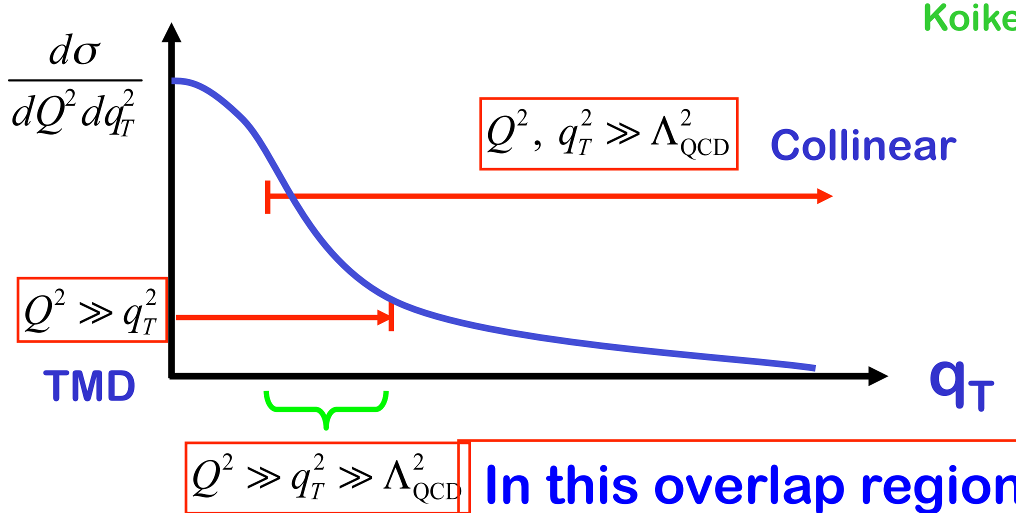
  Transverse-spin: $$\frac{1}{M_P}\int d^2k_\perp \, \vec{k}_\perp^2 \, q_T(x,k_\perp) + \mathrm{UVCT}(\mu_F^2) = T_F(x,x,\mu_F^2)$$

  But, TMD factorization is only valid for low k_T – TMD PDFs at low k_T

The consistency check

- IF both factorizations are proved to be valid,
  - ✧ both formalisms should yield the same result in overlap region
  - ✧ Case studies – Drell-Yan/SIDIS

Ji, Qiu, Vogelsang, and Yuan
Koike, Vogelsang, and Yuan



In this overlap region, both formalisms indeed give the same result

- TMD factorization fails for processes involving three or more identified hadrons!

Collins, Qiu, 2007
Vogelsang, Yuan, 2007, Collins, 2007
Rogers, Mulders, 2010

New challenges!

Collinear distributions

❑ Gauge link of collinear factorized distributions:

$$T(\{x_i\},\mu,S)=\int\prod_i^N \frac{dy_i^-}{2\pi}\, e^{ix_ip^+y_i^-}\,\langle p,S|\overline{\psi}(0)\gamma^+\,\boxed{\text{Gauge link}}\,\phi(y_i^-)\,\boxed{\text{Gauge link}}\,\psi(y_N^-)|p,S\rangle$$

All $\boxed{\text{Gauge link}}$ are on the same light-one with $y_i^+ = y_{i\perp} = 0$

❑ Parity and Time-reversal transformation:

$$\langle P,s_T|\hat{\mathcal{O}}(\psi,A_\mu)|P,s_T\rangle = \langle P,-s_T|\mathcal{PT}\hat{\mathcal{O}}(\psi,A_\mu)^\dagger\mathcal{T}^{-1}\mathcal{T}^{-1}|P,-s_T\rangle$$

$$\hat{\mathcal{O}}(\psi(y_i^-),A_\mu(y_j^-)) \;\Rightarrow\; \mathcal{PT}\,\hat{\mathcal{O}}(\psi(y_i^-),A_\mu(y_j^-))\,(\mathcal{PT})^{-1} \;\propto\; \hat{\mathcal{O}}(\psi(y_i^-),A_\mu(y_j^-))$$

✧ All collinear factorized distributions are process independent!

✧ The process dependence is included in perturbative coefficients

❑ Scheme dependence:

✧ Integration of kT into distributions $\Longrightarrow$ additional UV divergence

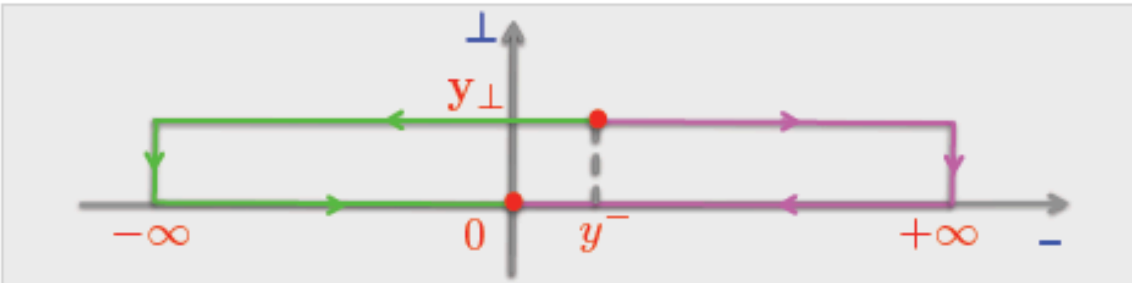
✧ Scheme dependence from the choice of UVCT(μ)

TMD parton distributions

❑ Gauge link dependence of TMD distributions:

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\,\boxed{\text{Gauge link}}\,\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

- SIDIS: $\Phi_n^\dagger(\{+\infty,0\},\mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(+\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\Phi_n(\{+\infty,y^-\},\mathbf{y}_\perp)$
- DY: $\Phi_n^\dagger(\{-\infty,0\},\mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(-\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\Phi_n(\{-\infty,y^-\},\mathbf{y}_\perp)$



Wilson Loop ~ $\exp\left[-ig\int_\Sigma d\sigma^{\mu\nu}F_{\mu\nu}\right]$ Area is NOT zero

- For a fixed spin state:

$$f_{q/h^\uparrow}^{\text{SIDIS}}(x,\mathbf{k}_\perp,\vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x,\mathbf{k}_\perp,\vec{S})$$

Modified universality of Sivers function

❑ Parity and Time-reversal for TMD operators:

$$\hat{\mathcal{O}}(\psi(y_i), A_\mu(y_j)) \quad \Rightarrow \quad \mathcal{PT}\,\hat{\mathcal{O}}(\psi(y_i), A_\mu(y_j))\,(\mathcal{PT})^{-1} \quad \cancel{\propto} \quad \hat{\mathcal{O}}(\psi(y_i), A_\mu(y_j))$$

$$\Longrightarrow \quad \langle p, S|\mathcal{O}_{q/h}^{\mathrm{SIDIS}}(\psi(y_i), A_\mu(y_j))|p, S\rangle \quad \neq \quad \pm\,\langle p, -S|\mathcal{O}_{q/h}^{\mathrm{SIDIS}}(\psi(y_i), A_\mu(y_j))|p, -S\rangle$$

❑ Modified universality:

$$\langle p, S|\mathcal{O}_{q/h}^{\mathrm{SIDIS}}(\psi(y_i), A_\mu(y_j))|p, S\rangle \quad = \quad \langle p, -S|\mathcal{O}_{q/h}^{\mathrm{DY}}(\psi(y_i), A_\mu(y_j))|p, -S\rangle$$

$$\Longrightarrow \quad A \quad \propto \quad \Big[\langle p, S|\mathcal{O}_{q/h}^{\mathrm{SIDIS}}(\psi(y_i), A_\mu(y_j))|p, S\rangle \; - \; \langle p, -S|\mathcal{O}_{q/h}^{\mathrm{SIDIS}}(\psi(y_i), A_\mu(y_j))|p, -S\rangle\Big]$$

$$= \quad -\Big[\langle p, S|\mathcal{O}_{q/h}^{\mathrm{DY}}(\psi(y_i), A_\mu(y_j))|p, S\rangle \; - \; \langle p, -S|\mathcal{O}_{q/h}^{\mathrm{DY}}(\psi(y_i), A_\mu(y_j))|p, -S\rangle\Big]$$

❑ Definition of Sivers function:

$$\mathcal{F}_{q/h}(x, k_T, s_T) \equiv \mathcal{F}_{q/h}(x, k_T) + f_{q/h}^{\mathrm{Sivers}}(x, k_T)\,\vec{s}_T \cdot (\hat{p} \times \hat{k}_T)$$

❑ The sign change – test of TMD factorization:

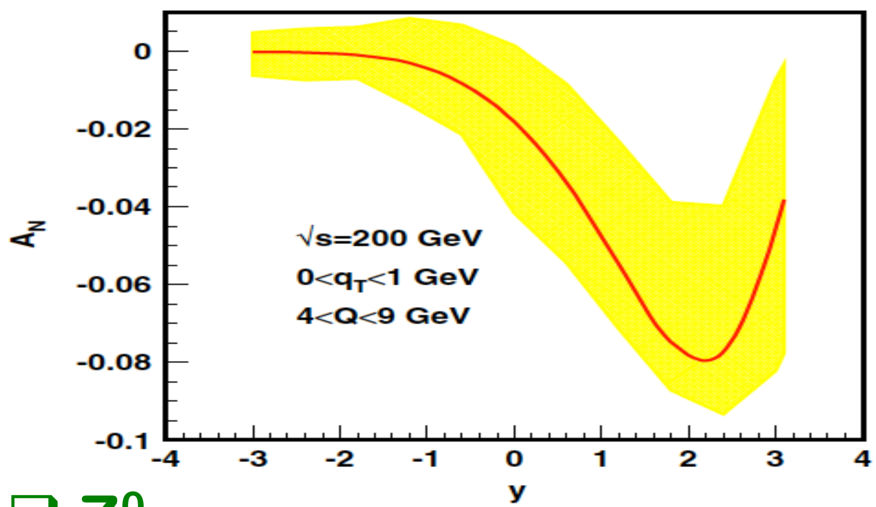
$$\mathcal{F}_{q/h}^{\mathrm{SIDIS}}(x, k_T, s_T) = \mathcal{F}_{q/h}^{DY}(x, k_T, -s_T)$$

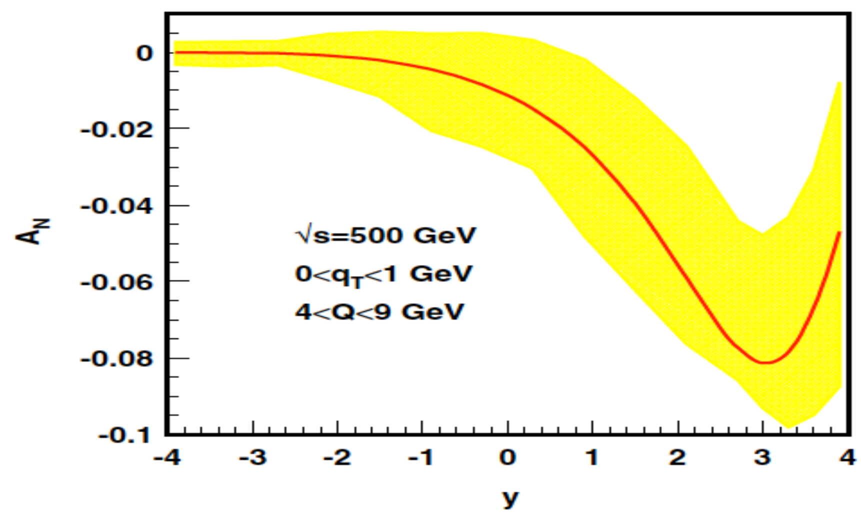
$$\Longrightarrow \quad f_{q/h^\uparrow}^{\mathrm{Sivers}}(x, k_\perp)^{\mathrm{SIDIS}} = -f_{q/h^\uparrow}^{\mathrm{Sivers}}(x, k_\perp)^{\mathrm{DY}}$$

Test of the modified universality

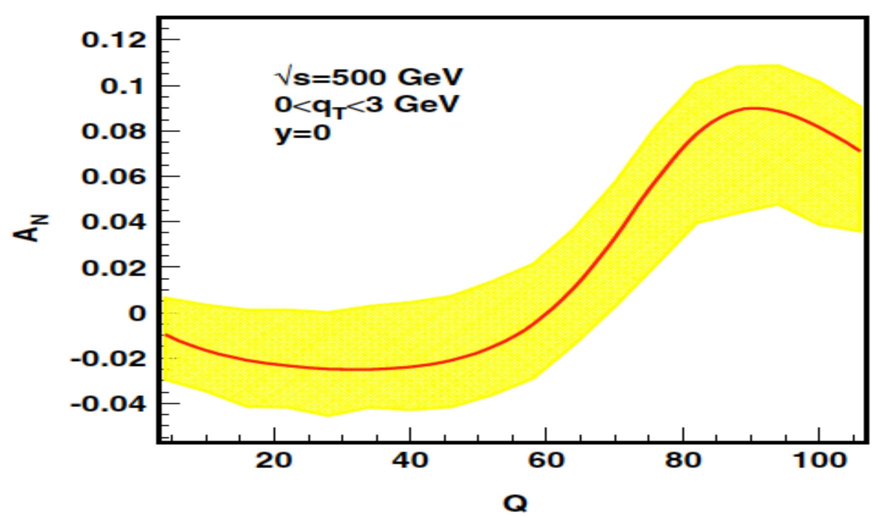
Collins et al. 2006
Kang, Qiu, 2009

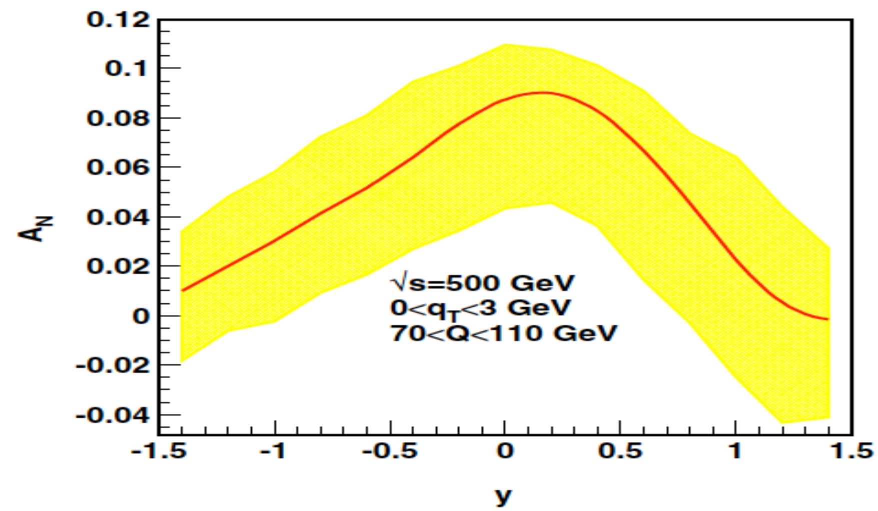
- Drell-Yan: $A_N^{\sin(\phi-\phi_s)} = -A_N$





- Z^0:

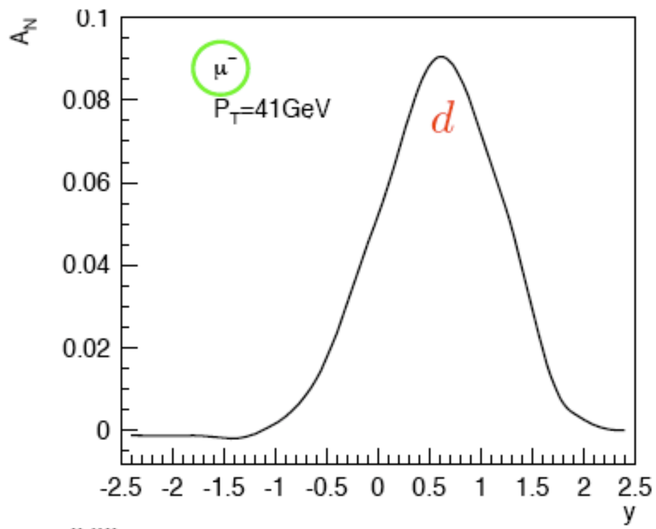


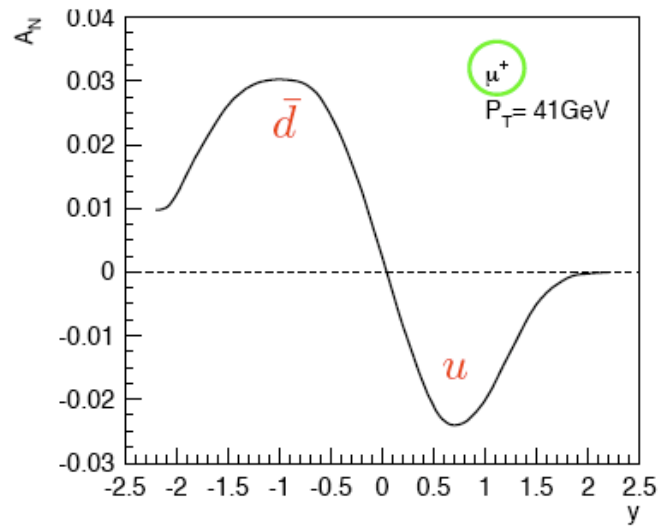


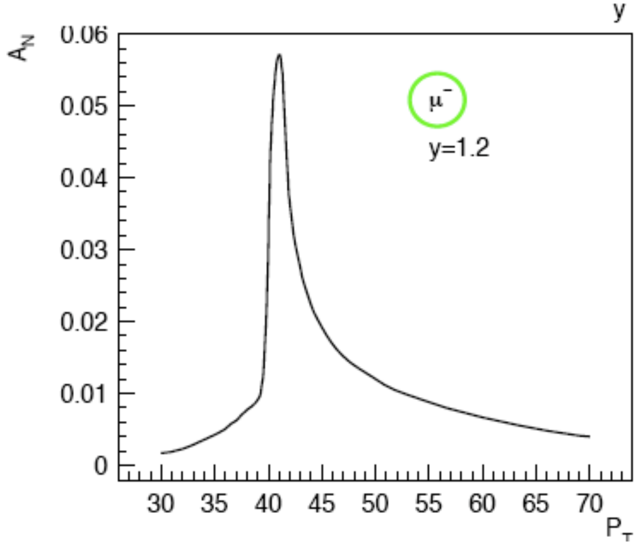
SSA of lepton from W-decay

Kang, Qiu, PRL 2009

- ❑ **Lepton SSA is diluted from the decay:**







- flavor separation
- asymmetry gets smaller due to dilution
  should still be measurable by current
  RHIC sensitivity

Complimentary to Drell-Yan/Z^0

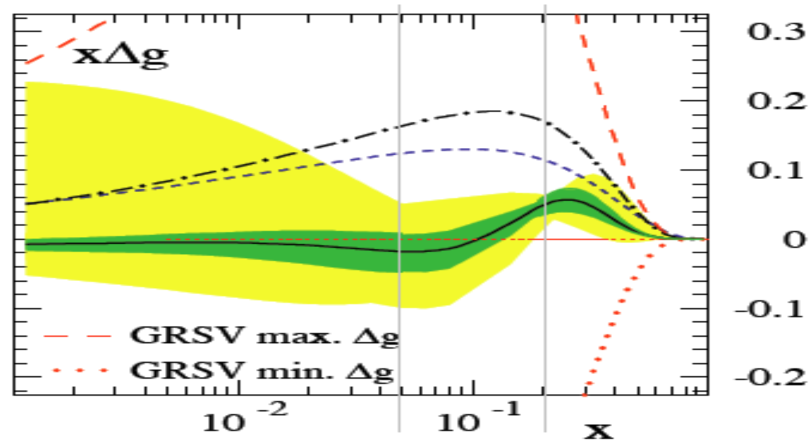
More see Kang's talk

One more caution on the sign change of A_N

- ❑ **Asymmetry could have a node:**

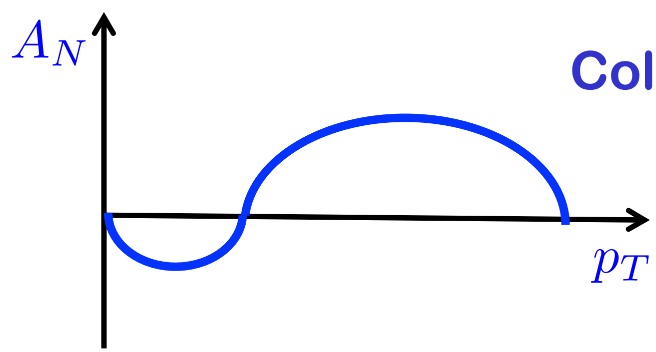
Sign change of Δg(x):

$$A(s) \propto \sigma(s) - \sigma(-s)$$



- ❑ **Asymmetry of Drell-Yan p_T distribution:**

We could have:



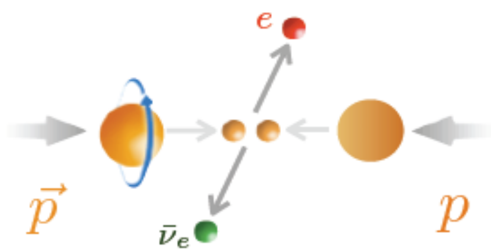
Collinear region (p_T ~ Q)

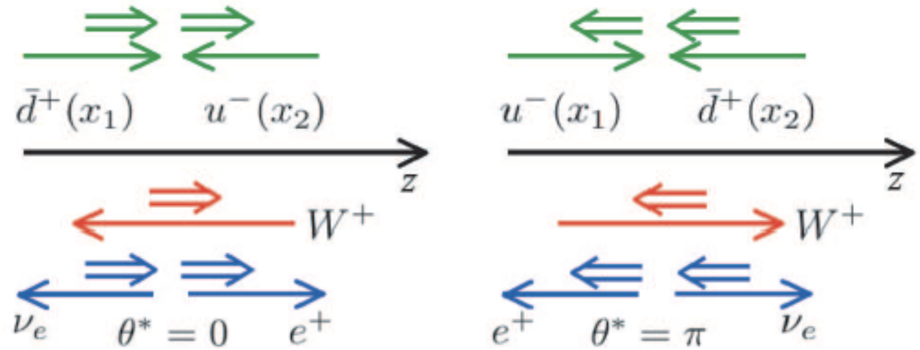
TMD region (p_T << Q)

Rich dynamics in p_T distribution or parton's transverse motion!

Drell-Yan with parity violation

- **W's are left-handed:**





- **Flavor separation:**

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}}e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}}e^{-y_W}$$

Forward W^+ (backward e^+):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

Backward W^+ (forward e^+):

$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

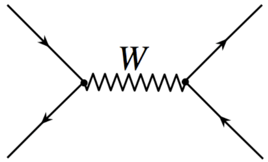
- **Complications:**

High order, W's p_T-distribution at low p_T

Challenge in predicting A_L of lepton

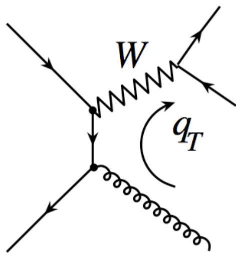
- RHIC experiments measure decay lepton not the W's:
- Fixed order pQCD calculation:

LO:



$$\propto \delta^2(q_T)$$

NLO:



$$\propto \frac{1}{q_T^2} \Rightarrow \infty \quad \text{as} \quad q_T^2 \to 0$$

Leptons not from W decay – background – hard for theorists

- All order resummation is needed:

CSS formalism – implemented in RHICBOS – only diagonal contribution

Resummation for the lepton angular distribution needed!

- Scale dependence:

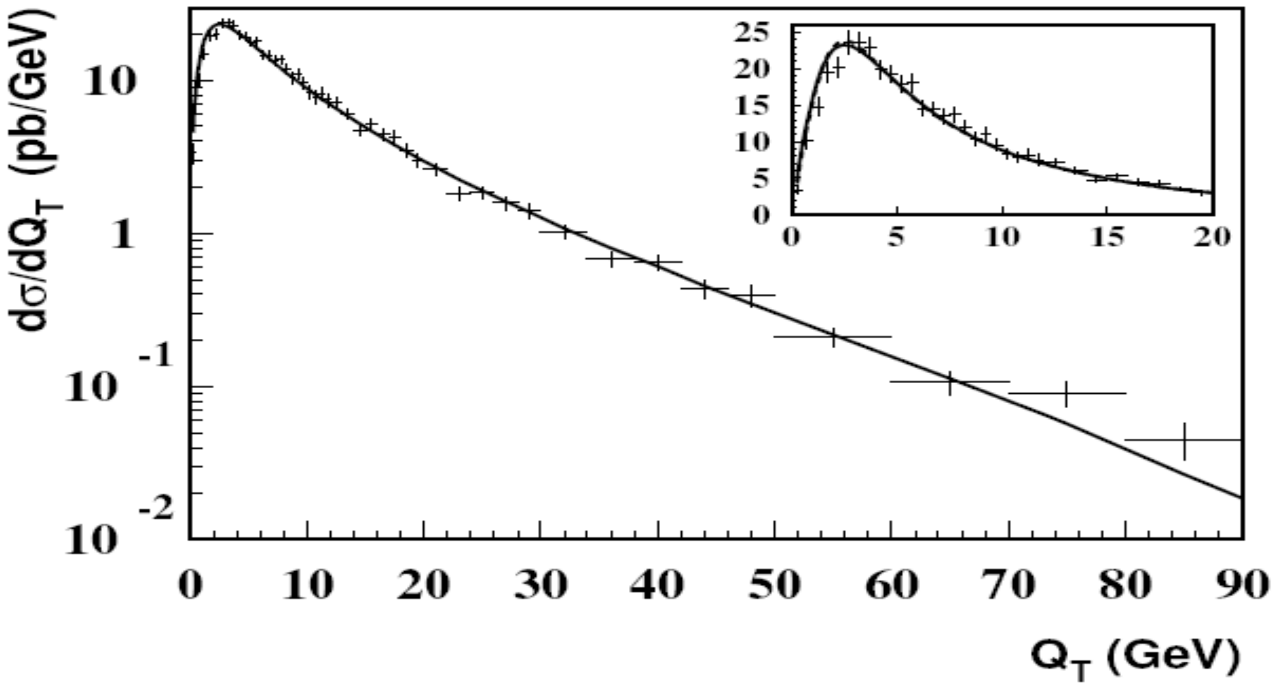
$$\Delta\bar{q}(\mu = M_W) \Longrightarrow \Delta\bar{q}(\mu = Q \sim \mathrm{GeV's})_{\mathrm{SIDIS}}$$

Unpolarized Drell-Yan cross section

- **The denominator of the Asymmetry:**

$$\frac{d\sigma}{d^4q} \qquad\qquad \frac{d\sigma}{d^4q\,d\Omega}$$

- **Angular integrated Drell-Yan is under control:**
  - Fermilab CDF data on Z at $\sqrt{S} = 1.8$ TeV



Unpolarized Drell-Yan cross section

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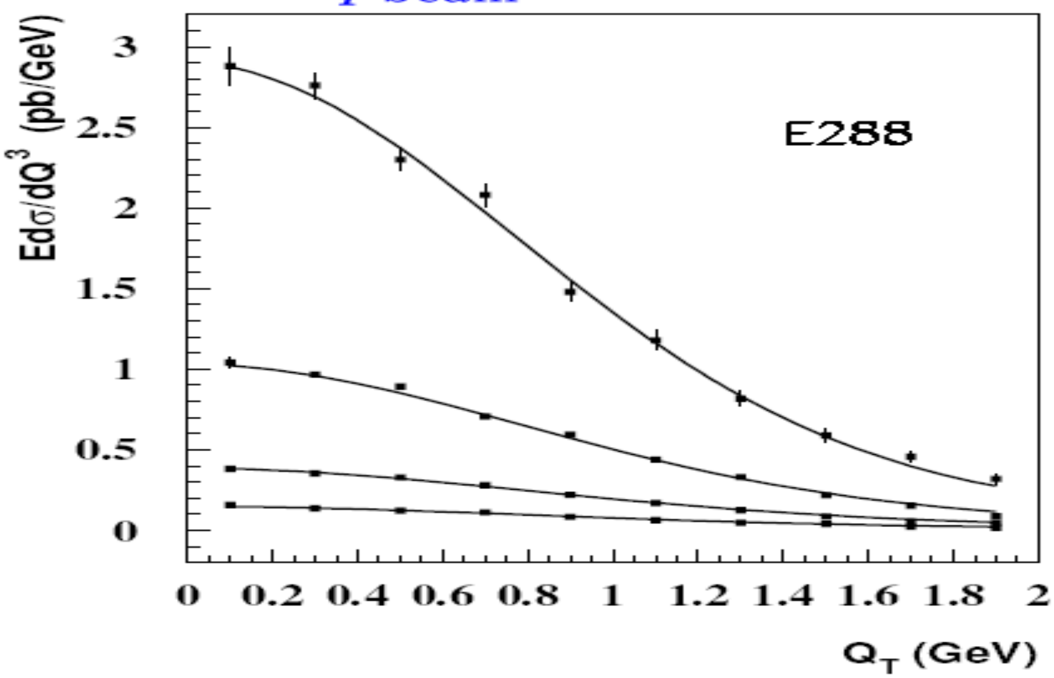

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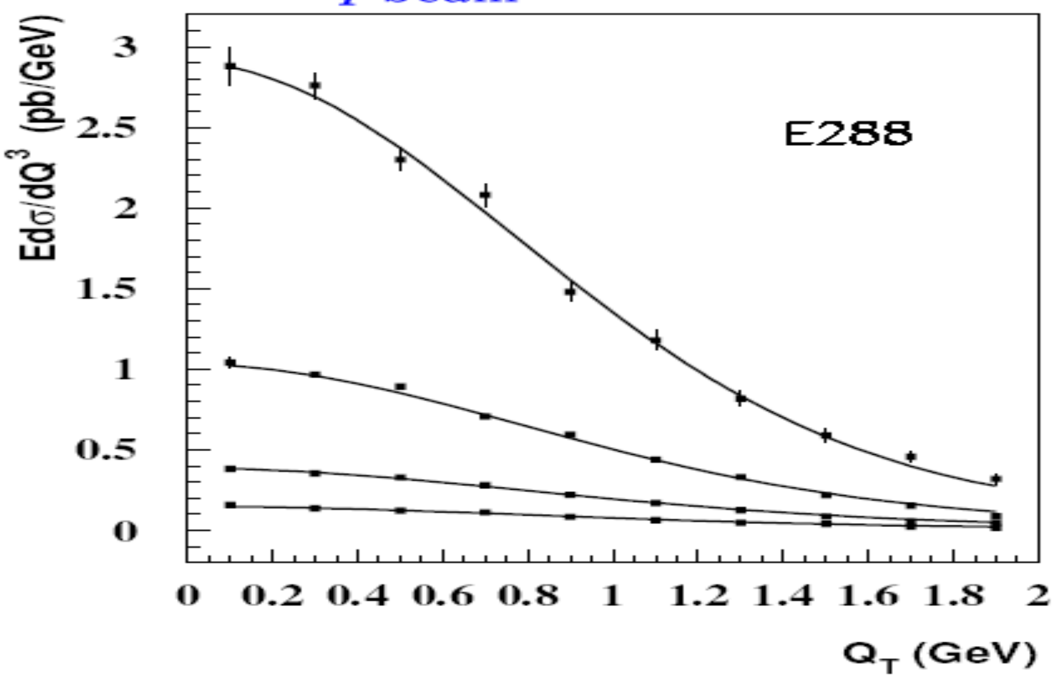


- But, Drell-Yan lepton angular distribution needs work!

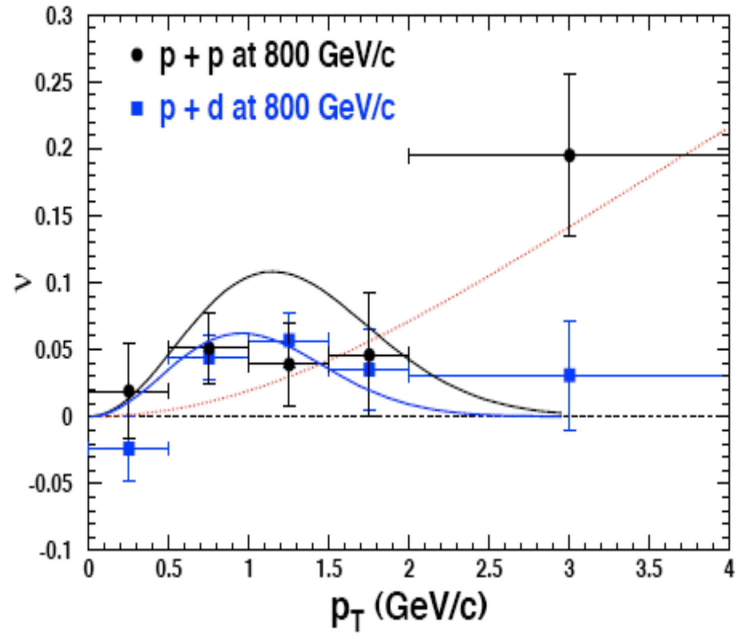
Violation of Lam-Tung relation

- **Normalized Drell-Yan lepton angular distribution:**

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{4\pi}\left(\frac{1}{\lambda+3}\right)\left[1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)\right]$$

- **Lam-Tung relation:**

$$1 - \lambda - 2\nu = 0$$



- **Collinear factorization:**

$$\lambda = \frac{W_T - W_L}{W_T + W_L} \approx \frac{W_T^{\rm Resum} - W_L^{\rm Resum}}{W_T^{\rm Resum} + W_L^{\rm Resum}} = \frac{1 - \frac{1}{2}Q_\perp^2/Q^2}{1 + \frac{3}{2}Q_\perp^2/Q^2}$$

$$\nu = \frac{2W_{\Delta\Delta}}{W_T + W_L} \approx \frac{2W_{\Delta\Delta}^{\rm Resum}}{W_T^{\rm Resum} + W_L^{\rm Resum}} = \frac{Q_\perp^2/Q^2}{1 + \frac{3}{2}Q_\perp^2/Q^2}$$

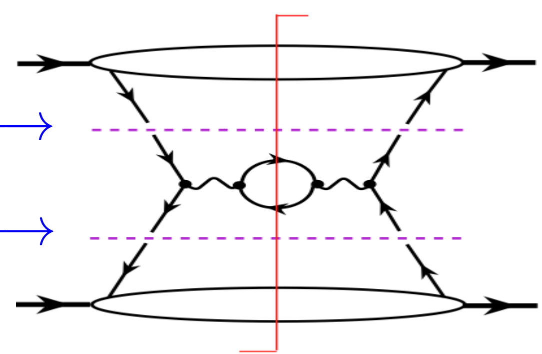
- **TMD factorization:**

  – **Boer – Mulder function:**

$$h_1^{\perp \rm DY}(x) = -h_1^{\perp \rm SIDIS}(x)$$

$\sigma^{+\alpha}k_{\perp\alpha} \longrightarrow$

$\sigma^{+\beta}k'_{\perp\beta} \longrightarrow$



Needs Collins function

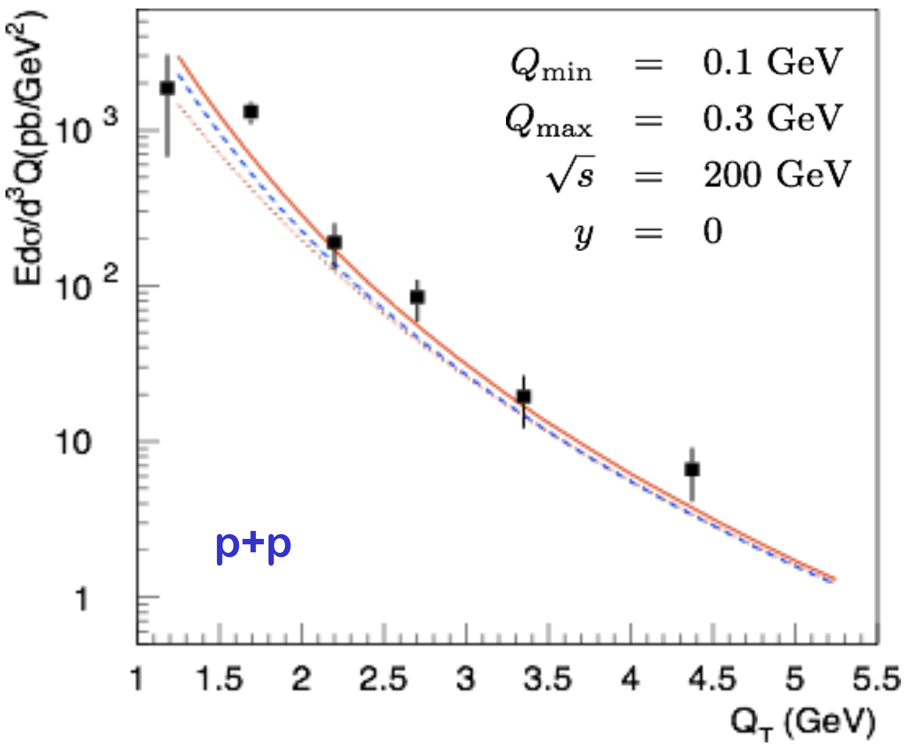
Low mass Drell-Yan ($p_T > Q$)

Kang, Qiu, Vogelsang, PRD 2009

❑ Invariant cross section:

$$E\frac{d\sigma_{AB\to\ell^+\ell^-(Q)X}}{d^3Q} \equiv \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2\, \frac{1}{\pi}\, \frac{d\sigma_{AB\to\ell^+\ell^-(Q)X}}{dQ^2\, dQ_T^2\, dy}$$

❑ Role of non-perturbative fragmentation function:



Data from PHENIX: arXiv:0804.4168

- ✧ **Input FF:**

  $$D(z,\mu_0) = D^{\rm QED}(z) + \kappa\, D^{\rm NP}(z)$$

- ✧ **QED alone (dotted):**

  $\kappa = 0$ at $\mu_0 = 1$ GeV

- ✧ **QED + hadronic input (solid):**

  $\kappa = 1$ at $\mu_0 = 1$ GeV

Hadronic component of fragmentation is very important at low Q_T

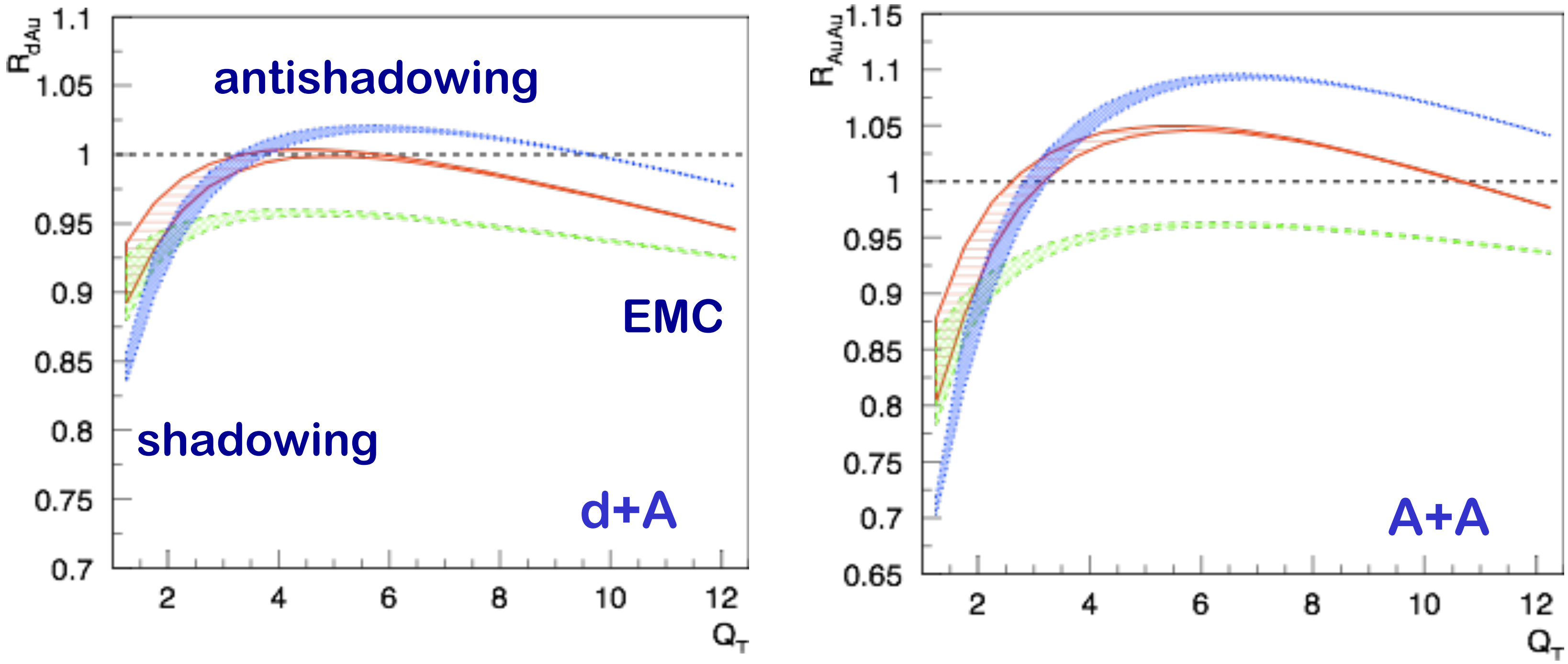
Excellent probe of gluon distribution

Kang, Qiu, Vogelsang, PRD 2009

- **Nuclear modification factor:**

$$R_{\mathrm{dAu}} \equiv \frac{1}{\langle N_{coll}\rangle}\frac{d^2N^{\mathrm{dAu}}/dQ_Tdy}{d^2N^{pp}/dQ_Tdy} \overset{\mathrm{min.bias}}{=} \frac{\frac{1}{2A}d^2\sigma^{\mathrm{dAu}}/dQ_Tdy}{d^2\sigma^{pp}/dQ_Tdy}$$

- **RHIC kinematics – if dominated by single scattering:**



– The band is given by κ=1 (top lines) and κ=0 (bottom lines)

– Ratio follows the feature of gluon distribution if turns off isospin

Summary and outlook

- Drell-Yan process is one of the oldest hard process proposed to test QCD – it still a very good one!

- The proof of QCD factorization for Drell-Yan is solid (LP + NLP for collinear, LP for TMD)

- The test of the sign change of the Sivers function is a critical test of TMD factorization!

- Drell-Yan could provide much more than the sign change

Thank you!

Backup transparencies